Work Energy Packet 3

Potential Energy and Conservation of Mechanical Energy

So far, we have mainly considered force(s) applied to a mass which moves over a straight line distance and the associated change in kinetic energy. But what happens when the displacement is not a straight line? Though still possible to use the Work Kinetic Energy Theorem, the constantly changing forces and directions rapidly make such problems extremely difficult. However, if we consider another type of energy called potential energy, we can make use of a concept called conservation of energy to easily solve these otherwise difficult problems.

What is potential energy?

Potential energy (PE) is energy associated with **position**. Recall, kinetic energy (KE) is energy associated with motion. For any mass m, moving at a velocity v, its kinetic energy is $\frac{1}{2} mv^2$. There are several types of potential energy in the universe. We will limit our discussion to gravitational potential energy, or energy associated with position in a gravitational field. That's a fancy way of saying energy associated with height.

Let's say you're trying to pound a wooden stake into the ground. Consider a big rock lying on the ground next to the stake compared to a big rock held up in the air above the stake. Recall, one description of energy is: "the ability to do work" which means the ability to exert a force over a distance. To pound the stake into the ground, you must exert a force on the stake over some distance, that is, do work on it. The rock on the ground won't help you pound in the stake. But if you can position the rock up in the air directly above the stake, and then let it go, the rock will fall onto the stake and exert a force on the stake, pushing it some distance into the ground. That is, the rock up in the air can do work on the stake. So the rock up in the air must have more energy than the rock on the ground. Initially, neither rock was moving, so both had zero kinetic energy. The additional energy of the rock up in the air is potential energy.

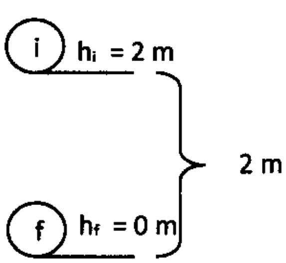
Potential energy of a mass m held a height Δh above the ground on earth is $PE = mg\Delta h$.

<u>Example 1</u>

What is the potential energy of a big rock with a mass of 12 Kg which is 2 m above the ground?

Solution:

The usual convention is to call the ground h=0. If you let go of the rock, it would go from $h_i = 2$ m to $h_f = 0$ m.



So: $\Delta h = (h_f - h_i)$

$\Delta h = (0 \text{ m} - 2 \text{ m})$

$\Delta h = -2 \text{ m}$

$PE = mg\Delta h$

$PE = (12 \text{ Kg})(-10 \text{ m/s}^2)(-2 \text{ m})$

$PE = 240 \text{ Kg m}^2/\text{s}^2$

which is the same as 240 J (a Kg m^2/s^2 is a Joule)

Most people don't mess with all the negative signs all the time. They keep track of the fact that the higher position has the higher PE and call $a_g = 10 \text{ m/s}^2$ and just say:

$$PE = mgh$$
$$PE = (12 \text{ Kg})(10 \text{ m/s}^2)(2 \text{ m})$$
$$PE = 240 \text{ J}$$

That's what I will do from now on.

Work due to gravity changes potential energy

How did the rock get from initial to final position? The force of gravity acted on it over a displacement of -2 m. So force of gravity ($F_g = ma_g$) did work (W) on the rock equal to the force times the displacement.

$W_g = F\cdot\Delta y$ (Where W_g is the work due to gravity, $F = ma_g$, and we use y instead of x since this is a vertical scenario)

$W_g = mg\Delta h$ (Where Δh = change in height = Δy)

Thus, the work due to gravity is the same as the potential energy term from page 1. Now, we know from the previous packet that a force acting in the direction of motion does positive work on an object, and that is the situation here. Gravity pulls down and the object moves down. But we also know from example 1, that as the rock goes down, its potential energy decreases by the amount $ma_g\Delta h$ as it falls from 2 m to earth. So we have gravity doing work equal to $ma_g\Delta h$ and the potential energy changing by $-ma_g\Delta h$. The result is:

$$W_g = -\Delta PE.$$

We know from the Work Energy Theorem of the previous packet that $W = \Delta KE$, and from above that $W_g = -\Delta PE$,

so plugging one into the other yields:

$$-\Delta PE = \Delta KE$$

rearranging gives

$$\Delta PE + \Delta KE = 0$$

Conservation of Mechanical Energy

This equation is deceptively simple. What it says is quite profound. It says that if a falling object's potential energy changes by some amount, its kinetic energy must change by an equal and opposite amount such that the sum of the changes comes to zero. In other words, in this situation, you can't create energy. You can just swap it back and forth between kinetic and potential. This concept is called "conservation of energy", meaning the total amount of energy is conserved. This is true in our specific example of a falling ball (caveat: you neglect air resistance.) Gravitational potential energy and kinetic energy are called mechanical energy, so this scenario demonstrates the conservation of mechanical energy in the absence of external forces other than gravity.

Another way to write $\Delta PE + \Delta KE = 0$ is as follows:

$$\Delta PE + \Delta KE = 0$$

$$(PE_f - PE_i) + (KE_f - KE_i) = 0$$

$$PE_i + KE_i = PE_f + KE_f$$ (in the absence of external forces other than gravity)

$$\text{Initial Energy } (E_i) = \text{Final Energy } (E_f)$$

Conservation of Energy

We have limited ourselves here to gravitational potential energy and kinetic energy and neglected friction. But when you include all types of energy in the universe, there are no caveats. A change in any one type of energy must be offset by an equal and opposite change in another type of energy. The total amount of energy in the universe hasn't changed since the big bang.

Other types of energy include: thermal energy (where work due to friction usually ends up), chemical potential energy (think of all the kinetic energy your car gets by snapping carbon bonds in gasoline) nuclear energy (almost all energy on earth ultimately comes from nuclear fusion in the sun), light, sound, and all the rest (in fact, Einstein showed mass to be energy in another form: $E = mc^2$. Energy equals mass times the speed of light squared)

Why is energy conserved?

I don't know. It just seems to be how the universe works. I've seen some math explanations that rely on arguments of mathematical symmetry to "prove" conservation of energy, but it seems to me that just peels the onion back by one layer. Why do those symmetries exist? Maybe one of you will go to college and figure it out. If you do, please explain it to me. For now, it falls in the category of: it is what it is, trust me.

When can you use Conservation of Mechanical Energy?

You can use Conservation of Mechanical Energy when the only work that is being done on the object comes from gravity. When that's the case, then the logic developed on the previous page of $W_g = -\Delta PE$ yields $E_i = E_f$. How can you tell whether the only work being done comes from gravity? Draw the FBD and initial final picture. Draw the direction of motion d. Do any trig as needed and redraw the FBD. Look at the force vectors on the redrawn FBD that are parallel to d. They are the only ones doing work on the object. (Why is that a true statement? Because the other vectors are perpendicular to d, and thus do no work...See Work Energy Packet 1 if needed). If F_g, F_{gX}, or F_{gY}, are the only force vectors on the redrawn FBD that are parallel to d, then gravity must be the only force doing work on the object and you can make use of Conservation of Mechanical Energy,

meaning $E_i = E_f$

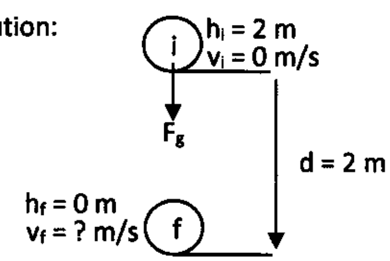
so $PE_i + KE_i = PE_f + KE_f$

and $mgh_i + ½\, m(v_i)^2 = mgh_f + ½\, m(v_f)^2$

<u>Example 2</u>

A ball is dropped from a height of 2 m. How fast is it going when it hits the ground? Neglect air resistance.

Solution:



> Draw the combined FBD initial / final drawing showing all forces and the displacement d. Do any trig if needed, and then think about the redrawn FBD along the path of motion as follows:
>
> **If the only force parallel to d is from gravity (meaning F_g, F_{gx}, F_{gy}) then mechanical energy is conserved and you may use $E_i = E_f$.**

We have done this problem in the beginning of the year using kinematics equations, and in the previous packet using the Work Energy Theorem, but the simplest way is to make use of conservation of mechanical energy as follows. Since the only force parallel to the displacement d comes from gravity, as described in the text box above, mechanical energy is conserved, meaning Initial Energy (E_i) = Final Energy (E_f)

$$E_i = E_f$$
$$PE_i + KE_i = PE_f + KE_f$$
$$mgh_i + ½\, m(v_i)^2 = mg\, h_f + ½\, m(v_f)^2$$

Notice that each term has an "m" mass term. If you divide the entire equation by "m", they will all drop out, leaving:

$$gh_i + ½\, (v_i)^2 = gh_f + ½\, (v_f)^2$$

$v_i = 0$ because the ball is "dropped" and $h_f = 0$ because the ground = 0 m

$$(10\ m/s^2)(2\ m) + ½\, (0\ m/s)^2 = (10\ m/s^2)(0\ m) + ½\, (v_f)^2$$
$$20 + 0 = 0 + ½\, (v_f)^2$$
$$(v_f)^2 = 40$$
$$v_f = 6.32\ m/s$$

When you get v from kinetic energy, you're only getting the speed. To know the direction and thus the velocity, you must consider the rest of the problem. Here, we can tell the ball is going straight down.

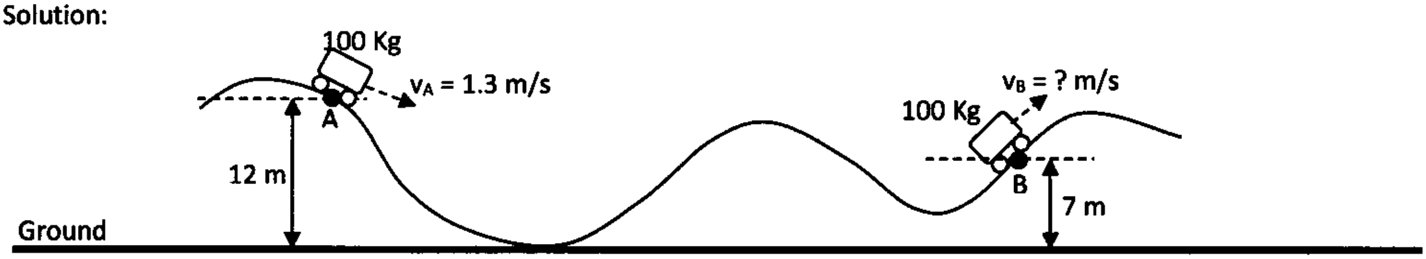
Notice, mass disappeared from this equation. This means the speed of a dropped ball has nothing to do with its mass (again, neglecting air resistance)

We could have solved the above situation with kinematics equations $[(v_{yf})^2 = (v_{yi})^2 + 2a_y\Delta y]$ or from the Work Kinetic Energy Theorem [$W_T = \Delta KE$ and W_T is work due to gravity] However, conservation of energy can solve problems much too complex for either of these two techniques as shown in the next example.

Example 3

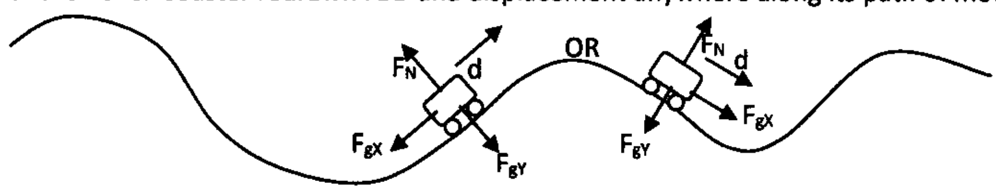
A 100 kg roller coaster travels from point A to point B. It is traveling at 1.3 m/s at point A. How fast is it traveling at point B? Neglect friction.

Solution:



This situation is extremely difficult to solve through Newton's law's and kinematics relationships. The path of the motion is irregular and the acceleration is non-constant, even neglecting friction. But it's quite easy with conservation of energy considerations.

Solution: Consider the Roller Coaster redrawn FBD and displacement anywhere along its path of motion as follows:



Even though all the forces and displacements are constantly changing, no matter where you consider the roller coaster, you get a FBD like one of the ones shown above. The only force acting on the roller coaster with a component parallel the direction of motion d is gravity (F_{gX}). F_N and F_{gY} always point exactly perpendicular to d, so neither does work on the roller coaster. This means mechanical energy is conserved, so you can proceed as follows:

$$E_i = E_f$$
$$PE_A + KE_A = PE_B + KE_B$$
$$mgh_A + ½\, m(v_A)^2 = mgh_B + ½\, m(v_B)^2$$
$$gh_A + ½\, (v_A)^2 = gh_B + ½\, (v_B)^2$$

$v_A = 1.3$ m/s, $h_A = 12$ m, $h_B = 7$ m

$$(10\text{ m/s}^2)(12\text{ m}) + ½\,(1.3\text{ m/s})^2 = (10\text{ m/s}^2)(7\text{ m}) + ½\,(v_f)^2$$
$$120 + 0.85 = 70 + ½\,(v_f)^2$$
$$(v_f)^2 = 101.7$$
$$v_f = 10.08\text{ m/s}$$

Meaning 10.08 m/s is the magnitude of velocity. Its direction is parallel to the direction of the tracks at point B. We don't have any more detail than that to specify an actual angle.

Work Energy 3

Practice Questions

Use conservation of mechanical energy concepts, not kinematics. Neglect air resistance and friction

1. What is the potential energy of a 0.3 kg book which you hold at 1.5 m above the ground?

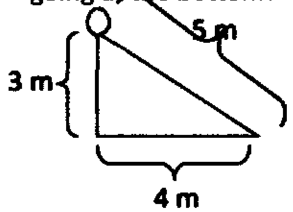
2. You drop a 5 kg ball from a 4th floor window which is 12 m above the ground. How fast is the ball going just before it hits the ground?

3. You drop a 17 kg ball from a 4th floor window which is 12 m above the ground. How fast is the ball going just before it hits the ground?

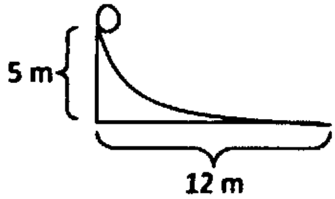
4. You drop a ball of unknown mass from a 4th floor window which is 12 m above the ground.
   a. How fast is the ball going just before it hits the ground?

   b. Same scenario as above. How fast is the ball going as it passes the 2nd floor window, which is 6 m above the ground?

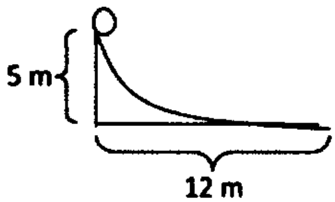
5. You slide an object down a ramp as shown. Neglect friction. It starts at the top from rest. How fast is the object going at the bottom?



6. You slide an object down a curved ramp as shown. Neglect friction. It starts at the top from rest. How fast is the object going at the bottom?



7. You slide an object down a curved ramp as shown. Neglect friction. It starts at the top with an initial speed of 4 m/s. How fast is the object going at the bottom?



8. You throw a ball of unknown mass from a 4th floor window which is 12 m above the ground with an initial velocity of 5 m/s.
    a. How fast is the ball going just before it hits the ground?

    b. Same scenario as above. How fast is the ball going as it passes the 2nd floor window, which is 6 m above the ground? (answer: 12.04 m/s)

    c. Does you answer to a and b above depend on if the ball is thrown straight up as compared to horizontally? Why or why not?

9. You drop a penny down a wishing well. It is falling at a velocity of 25 m/s as it enters the water at the bottom of the well. How deep is the well?

10. You throw a penny straight down a wishing well with an initial velocity of 3 m/s. It is falling at a velocity of 20 m/s as it enters the water at the bottom of the well. How deep is the well?

11. A roller coaster passes the highest point on its track at a speed of 3 m/s. This point is 30 m in the air. Neglecting friction and air resistance:
    a. How fast is it going at the bottom of the track which is at ground level?

    b. Same scenario as above. How fast is it going at the top of the next hill, which is 25 m in the air?

Answers

1. 4.5 J
2. 15.49 m/s
3. 15.49 m/s
4. a) 15.49 m/s b) 10.95 m/s
5. 7.75 m/s
6. 10 m/s
7. 10.77 m/s
8. a) 16.28 m/s b) 12.04 m/s
9. 31.25 m
10. 19.55 m
11. a) 24.68 m/s) b) 10.44 m/s